# Trigonometric identities

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## Contents

# 1 Algebra

## 1.1 Definitions

$$\sin x = \frac{1}{\csc x} = \frac{e^{ix} - e^{-ix}}{2i} \qquad \sinh x = \frac{1}{\operatorname{csch} x} = \frac{e^{x} - e^{-x}}{2}$$

$$\cos x = \frac{1}{\sec x} = \frac{e^{ix} + e^{-ix}}{2} \qquad \cosh x = \frac{1}{\operatorname{sech} x} = \frac{e^{x} + e^{-x}}{2}$$

$$\tan x = \frac{1}{\cot x} = -i\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \qquad \tanh x = \frac{1}{\coth x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

## 1.2 Quotient identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

## 1.3 Hyperbolic relations

$$\sinh x = -i\sin(ix) \qquad \cosh x = \cos(ix) \qquad \tanh x = -i\tan(ix)$$

$$\operatorname{csch} x = i\csc(ix) \qquad \operatorname{sech} x = \sec(ix) \qquad \coth x = i\cot(ix)$$

## 1.4 Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sec^2 x - \tan^2 x = 1 \qquad \csc^2 x - \cot^2 x = 1$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \operatorname{sech}^2 x + \tanh^2 x = 1 \qquad \coth^2 x - \operatorname{csch}^2 x = 1$$

## 1.5 Co-function identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

## 1.6 Parity relations

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh x \qquad \tanh(-x) = -\tanh x$$

$$\operatorname{csch}(-x) = -\operatorname{csch} x \qquad \operatorname{sech}(-x) = \operatorname{sech} x \qquad \coth(-x) = -\coth x$$

## 1.7 Sum-difference formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad \sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y} \qquad \tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

## 1.8 Double angle formulas

$$\sin(2x) = 2\sin x \cos x \qquad \sinh(2x) = 2\sinh x \cosh x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x} \qquad \tanh(2x) = \frac{2\tanh x}{1 + \tanh^2 x}$$

## 1.9 Power-reducing, half angle formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2} \qquad \tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

$$\sinh^2 x = \frac{\cosh(2x) - 1}{2} \qquad \cosh^2 x = \frac{\cosh(2x) + 1}{2} \qquad \tanh^2 x = \frac{\cosh(2x) - 1}{\cosh(2x) - 1}$$

## 1.10 Sum to product formulas

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

## 1.11 Product to sum formulas

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right] \qquad \cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

## 1.12 Euler equations

$$e^x = \cosh x + \sinh x \qquad e^{-x} = \cosh x - \sinh x$$

$$e^{ix} = \cos x + i\sin x \qquad e^{-ix} = \cos x - i\sin x$$

## 1.13 Inverse relations

These are derivable from the co-function identities.

$$\cos^{-1} = \frac{\pi}{2} - \sin^{-1} x \qquad \csc^{-1} = \frac{\pi}{2} - \sec^{-1} x \qquad \cot^{-1} = \frac{\pi}{2} - \tan^{-1} x$$

$$\sin^{-1}(-x) = -\sin^{-1} x \qquad \cos^{-1}(-x) = \pi - \cos^{-1} x \qquad \tan^{-1}(-x) = -\tan^{-1} x$$

$$\csc^{-1}(-x) = -\csc^{-1} x \qquad \sec^{-1}(-x) = \pi - \sec^{-1} x \qquad \cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$\cos^{-1}\frac{1}{x} = \sec^{-1} x \qquad \sin^{-1}\frac{1}{x} = \csc^{-1} x \qquad \sec^{-1}\frac{1}{x} = \cos^{-1} x \qquad \csc^{-1}\frac{1}{x} = \sin^{-1} x$$

$$\begin{aligned}
\tan^{-1}\frac{1}{x} &= \frac{\pi}{2} - \tan^{-1} x = \cot^{-1} x && x > 0 \\
\tan^{-1}\frac{1}{x} &= -\frac{\pi}{2} - \tan^{-1} x = -\pi + \cot^{-1} x && x < 0 \\
\cot^{-1}\frac{1}{x} &= \frac{\pi}{2} - \cot^{-1} x = \tan^{-1} x && x > 0 \\
\cot^{-1}\frac{1}{x} &= \frac{3\pi}{2} - \cot^{-1} x = \pi + \tan^{-1} x && x < 0
\end{aligned}$$

These are derivable from the half-angle formulas:

$$\begin{aligned}
\sin^{-1} x &= 2\tan^{-1}\frac{x}{1+\sqrt{1-x^2}} \\
\cos^{-1} x &= 2\tan^{-1}\frac{\sqrt{1-x^2}}{1+x} && -1 < x \le 1 \\
\tan^{-1} x &= 2\tan^{-1}\frac{x}{1+\sqrt{1+x^2}}
\end{aligned}$$

Similarly, a few for hyperbolic functions are

$$\operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x} \qquad \operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x} \qquad \coth^{-1} x = \tanh^{-1}\frac{1}{x}$$

## 1.14 Inverse functions as logarithms

Note: give branch cut locations.

$$\sin^{-1} x = -i \ln\left(ix + \sqrt{1-x^2}\right) \qquad |x| < 1$$

$$\cos^{-1} x = -i \ln\left(x + \sqrt{x^2-1}\right) \qquad |x| < 1$$

$$\tan^{-1} x = \frac{i}{2} \ln \frac{1-ix}{1+ix}$$

$$\csc^{-1} x = -i \ln\left(\frac{i}{x} + \sqrt{\frac{x^2-1}{x^2}}\right) \qquad |x| \geq 1$$

$$\sec^{-1} x = -i \ln\left(\frac{1}{x} + \sqrt{\frac{1-x^2}{x^2}}\right) \qquad |x| \geq 1$$

$$\cot^{-1} x = \frac{i}{2} \ln \frac{x-i}{x+i}$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x-1}\sqrt{x+1}\right) \qquad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x} \qquad |x| < 1$$

$$\operatorname{sech}^{-1} x = \ln\left(x^{-1} + \sqrt{x^{-1}-1}\sqrt{x^{-1}+1}\right) \qquad 0 < x \leq 1$$

$$\operatorname{csch}^{-1} x = \ln\left(x^{-1} + \sqrt{1+x^{-2}}\right)$$

$$\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1} \qquad |x| < 1$$

Simplifications of root products are not made since we assume principal roots (this is irrelevant for real $x$). Note that $\operatorname{csch}^{-1} 2 = \ln \frac{1+\sqrt{5}}{2}$.

## 1.15 Inverse sum formulas

This is derivable from the tangent sum formula.

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

## 1.16 Triangle laws

These are for any triangle with sides of length $a$, $b$, and $c$ with angles opposite those sides $A$, $B$, $C$.

### Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$$

where $R$ is the circumradius.

**Law of cosines**

$$c^2 = a^2 + b^2 - 2ab\cos C$$

**Law of tangents**

$$\frac{a+b}{a-b} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$

### 1.17 Tangent formulas

$$\begin{aligned}
\tan(nx) &= \frac{\tan[(n-1)x] + \tan x}{1 - \tan[(n-1)x]\tan x} \\
\tan\frac{x}{2} &= \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x} = \frac{\tan x \sin x}{\tan x + \sin x} \\
\cos x &= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\
\sin x &= \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\
\tan\left(x+\frac{\pi}{4}\right) &= \frac{1+\tan x}{1-\tan x}
\end{aligned}$$

## 2 Calculus

### 2.1 Series

$$\begin{aligned}
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots & &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots & &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots & &= \sum_{n=0}^{\infty} \frac{U_{2n+1} x^{2n+1}}{(2n+1)!} \\
& & &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n}\left(2^{2n}-1\right) B_{2n} x^{2n-1}}{(2n)!} & |x| < \frac{\pi}{2} \\
\csc x &= \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \dots & &= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2\left(2^{2n-1}-1\right) B_{2n} x^{2n-1}}{(2n)!} & 0 < |x| < \pi \\
\sec x &= 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots & &= \sum_{n=0}^{\infty} \frac{U_{2n} x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n} x^{2n}}{(2n)!} & |x| < \frac{\pi}{2} \\
\cot x &= \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \dots & &= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} B_{2n} x^{2n-1}}{(2n)!} & 0 < |x| < \pi
\end{aligned}$$

where $U_n$ is the $n$th up/down number, $B_n$ is the $n$th Bernoulli number, and $E_n$ is the $n$th Euler number.

## 2.2 Inverse series

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| \le 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \left[x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| \le 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad |x| \le 1, \; x \neq i, -i$$

$$\csc^{-1} x = x^{-1} + \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| \ge 1$$

$$\sec^{-1} x = \frac{\pi}{2} - \left[x^{-1} + \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| \ge 1$$

$$\cot^{-1} z = \frac{\pi}{2} - \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{2n+1} \qquad |x| \le 1, \; x \neq i, -i$$

A more efficient series for inverse tangent:

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{2^{2n}(n!)^2}{(2n+1)!}\frac{x^{2n+1}}{(1+x^2)^{n+1}}$$

## 2.3 Hyperbolic series

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots = \sum_{n=1}^{\infty} \frac{2^{2n}\left(2^{2n}-1\right)B_{2n}x^{2n-1}}{(2n)!} \qquad |x| < \frac{\pi}{2}$$

$$\operatorname{csch} x = \frac{1}{x} - \frac{x}{6} + \frac{7x^3}{360} - \frac{31x^5}{15120} + \dots = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2\left(1-2^{2n-1}\right)B_{2n}x^{2n-1}}{(2n)!} \qquad 0 < |x| < \pi$$

$$\operatorname{sech} x = 1 - \frac{x^2}{2} + \frac{5x^4}{24} - \frac{61x^6}{720} + \dots = \sum_{n=0}^{\infty} \frac{E_{2n}x^{2n}}{(2n)!} \qquad |x| < \frac{\pi}{2}$$

$$\coth x = \frac{1}{x} + \frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \dots = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2^{2n}B_{2n}x^{2n-1}}{(2n)!} \qquad 0 < |x| < \pi$$

where $B_n$ is the $n$th Bernoulli number, and $E_n$ is the $n$th Euler number.

## 2.4 Inverse hyperbolic series

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| < 1$$

$$\cosh^{-1} x = \ln(2x) - \left[\frac{1}{2}\frac{x^{-2}}{2} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-4}}{4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-6}}{6} + \ldots\right] = \ln 2x - \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{-2n}}{2n} \qquad x > 1$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \qquad |x| < 1$$

$$\operatorname{csch}^{-1} x = x^{-1} - \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| < 1$$

$$\operatorname{sech}^{-1} x = \ln\frac{2}{x} - \left[\frac{1}{2}\frac{x^2}{2} + \frac{1\cdot 3}{2\cdot 4}\frac{x^4}{4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^6}{6} + \ldots\right] = \ln\frac{2}{x} - \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{2n}}{2n} \qquad 0 < x \le 1$$

$$\coth^{-1} x = x^{-1} + \frac{x^{-3}}{3} + \frac{x^{-5}}{5} + \frac{x^{-7}}{7} + \ldots = \sum_{n=0}^{\infty} \frac{x^{-(2n+1)}}{2n+1} \qquad |x| > 1$$

Expanding about $\infty$,

$$\sinh^{-1} x = \ln 2 - \ln x^{-1} + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)!!}{2n\,(2n)!!} x^{-2n}$$

$$\cosh^{-1} x = \ln 2 - \ln x^{-1} - \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2n(2n)!!} x^{-2n}$$

$$\operatorname{csch}^{-1} x = \sum_{n=1}^{\infty} \frac{P_{n-1}(0)}{n} x^{-n}$$

$$\operatorname{sech}^{-1} x = \ln 2 - \ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)!!}{2n(2n)!!} x^{2n}$$

$$\coth^{-1} x =$$

where $n!! = n\cdot(n-2)\cdot(n-4)\ldots$ down to 1 or 2 if $n$ is odd or even. $P_n(x)$ is a Legendre polynomial.

## 2.5 Derivatives

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

## 2.6 Inverse derivatives

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1} x = -\frac{1}{x^2\sqrt{1-x^{-2}}} \qquad \frac{d}{dx}\sec^{-1} x = \frac{1}{x^2\sqrt{1-x^{-2}}} \qquad \frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$

## 2.7 Hyperbolic derivatives

$$\frac{d}{dx}\sinh x = \cosh x \qquad \frac{d}{dx}\cosh x = \sinh x \qquad \frac{d}{dx}\tanh x = \operatorname{sech}^2 x$$
$$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x \qquad \frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

## 2.8 Inverse hyperbolic derivatives

$$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x-1}\sqrt{x+1}} \qquad \frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$$
$$\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{x^2\sqrt{1+x^{-2}}} \qquad \frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x(x+1)\sqrt{\frac{1-x}{1+x}}} \qquad \frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}$$

## 2.9 Integrals

$$\int \sin x = -\cos x \qquad \int \cos x = \sin x \qquad \int \tan x = -\ln|\cos x|$$
$$\int \csc x = -\ln|\csc x + \cot x| \qquad \int \sec x = \ln|\sec x + \tan x| \qquad \int \cot x = \ln|\sin x|$$

## 2.10 Inverse integrals

Compute by integration by parts using the derivative of the function.

## 2.11 Hyperbolic integrals

$$\int \sinh x = \cosh x \qquad \int \cosh x = \sinh x \qquad \int \tanh x = \ln|\cosh x|$$
$$\int \operatorname{csch} x = \ln\left|\tanh\frac{x}{2}\right| \qquad \int \operatorname{sech} x = 2\tan^{-1}\tanh\frac{x}{2} \qquad \int \coth x = \ln|\sinh x|$$

## 2.12 Inverse hyperbolic integrals

Compute by integration by parts using the derivative of the function.