# Scattering and transfer matrices

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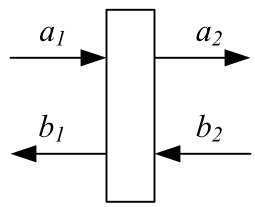


$$\begin{bmatrix} b_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ b_2 \end{bmatrix} \qquad \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \tag{1}$$

Multiplying these out gives

$$a_2 = T_{aa}a_1 + T_{ab}b_1 \tag{2}$$
$$b_2 = T_{ba}a_1 + T_{bb}b_1 \tag{3}$$
$$b_1 = S_{11}a_1 + S_{12}b_2 \tag{4}$$
$$a_2 = S_{21}a_1 + S_{22}b_2 \tag{5}$$

Substiting the third in the first to eliminate $b_1$ and matching with the fourth gives

$$a_2 = (T_{aa} + T_{ab}S_{11})a_1 + T_{ab}S_{12}b_2 = S_{21}a_1 + S_{22}b_2$$

Substituting the third in the second to eliminate $b_1$:

$$b_2 = T_{ba}a_1 + T_{bb}S_{11}a_1 + T_{bb}S_{12}b_2$$

These must hold for any $a_1$ and $b_2$, so we arrive at the relations

$$T_{aa} + T_{ab}S_{11} = S_{21} \tag{6}$$
$$T_{ab}S_{12} = S_{22} \tag{7}$$
$$T_{ba} + T_{bb}S_{11} = 0 \tag{8}$$
$$T_{bb}S_{12} = I \tag{9}$$

We can then derive the following conversions:

$$T = \begin{bmatrix} S_{21} - S_{22}S_{12}^{-1}S_{11} & S_{22}S_{12}^{-1} \\ -S_{12}^{-1}S_{11} & S_{12}^{-1} \end{bmatrix} \qquad S = \begin{bmatrix} -T_{bb}^{-1}T_{ba} & T_{bb}^{-1} \\ T_{aa} - T_{ab}T_{bb}^{-1}T_{ba} & T_{ab}T_{bb}^{-1} \end{bmatrix} \tag{10}$$