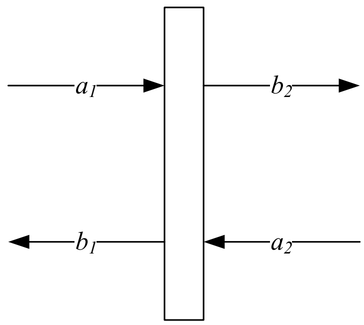

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = \begin{bmatrix} T_{ba} & T_{bb} \\ T_{aa} & T_{ab} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \tag{2}$$

Multiplying these out gives

$$b_2 = T_{ba}a_1 + T_{bb}b_1 \tag{3}$$
$$a_2 = T_{aa}a_1 + T_{ab}b_1 \tag{4}$$
$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{5}$$
$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{6}$$

Substiting the third in the first to eliminate b_1 and matching with the fourth gives

$$b_2 = (T_{ba} + T_{bb}S_{11})a_1 + T_{bb}S_{12}a_2 = S_{21}a_1 + S_{22}a_2$$

Substituting the third in the second to eliminate b_1:

$$a_2 = T_{aa}a_1 + T_{ab}S_{11}a_1 + T_{ab}S_{12}a_2$$

These must hold for any a_1 and a_2, so we arrive at the relations

$$T_{ba} + T_{bb}S_{11} = S_{21} \tag{7}$$
$$T_{bb}S_{12} = S_{22} \tag{8}$$
$$T_{aa} + T_{ab}S_{11} = 0 \tag{9}$$
$$T_{ab}S_{12} = I \tag{10}$$

We can then derive the following conversions:

$$T = \begin{bmatrix} S_{21} - S_{22}S_{12}^{-1}S_{11} & S_{22}S_{12}^{-1} \\ -S_{12}^{-1}S_{11} & S_{12}^{-1} \end{bmatrix} \tag{11}$$

$$S = \begin{bmatrix} -T_{ab}^{-1}T_{aa} & T_{ab}^{-1} \\ T_{ba} - T_{bb}T_{ab}^{-1}T_{aa} & T_{bb}T_{ab}^{-1} \end{bmatrix} \tag{12}$$